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Technical Notes

THE CORRECT SPECIFICATION OF OPPORTUNITY COST IN THE EVALUATION OF BOND ISSUE SIZE AND TIMING

John A. Halloran and Howard P. Lanser
University of Notre Dame

ABSTRACT

The question of when and in what quantity to obtain required bond financing takes on significance due to fixed flotation costs and the opportunity to invest excess long-term debt in marketable securities. In prior research, the optimal size and timing of bond issues is most often determined as the solution to a cost minimization problem. In this paper, we first demonstrate that previous studies have incorrectly identified the opportunity cost of excess debt. We then delineate the opportunity cost in a form consistent with modern capital structure theory. Our results indicate that a firm's capital structure and marginal tax rate are important determinants of the optimal size and timing of its bond issues.

The issue of the optimal capital structure for a firm has served as the focus of much research over the last three decades. In particular, Modigliani and Miller [9] sparked a debate concerning the valuation effect of debt financing that continues. Research into the issue of the optimal **level** of debt financing has raised additional questions concerning the effect of the **maturity composition** of the debt and the **timing** of its issuance on the value of the firm. This latter question is the topic of this paper.

Regardless of the specific model or approach used, research into the optimal size and timing of bond issues requires accurate identification and estimation of the opportunity cost of carrying excess debt financing. Many prior studies, which applied the familiar *EOQ* model, misspecified carrying costs by concentrating on interest rate differentials between long-term debt and short-term marketable securities. Our goal is to provide an alternate specification of carrying costs that reflects the insight provided by Modigliani and Miller regarding the valuation effect of borrowing and lending by a firm.

THE COSTS OF DEBT ISSUANCE AS IDENTIFIED IN PRIOR RESEARCH

Determining the optimal size and frequency of debt issuance requires the evaluation of the tradeoff between out-of-the-pocket flotation costs and the opportunity costs of carrying excess debt. One of the earliest attempts to evaluate this tradeoff was made by Bierman and McAdams (*B&M*) [1] who assumed that the firm faces the choice of acquiring the financing through one large debt issue or through a number of small sequential issues. The total flotation costs were divided into fixed and variable components:

$$\text{Flotation Costs} = F\left(\frac{D}{S}\right) + VD \tag{1}$$

where F is the fixed flotation cost per debt issue; D is the total debt financing requirement; S is the dollar size of each bond issue; and V is the variable flotation cost per dollar of total debt required. Equation (1) implies that flotation costs will be minimized when $D/S = 1$ (i.e., by utilizing one large debt issue). However, one large debt issue will result in a temporary surplus of available debt financing. If this surplus is temporarily invested in short-term marketable securities, it carries an opportunity cost equal to the difference in interest expense on long-term debt and interest revenues on short-term securities. In order to estimate the opportunity cost of excess debt, *B&M* assumed that a firm's cumulative financing requirement increased at a uniform rate over time. Since debt financing is acquired periodically in increments equal to S, average excess financing between bond issues and over the entire planning period will equal $S/2$ assuming a uniform rate of financing requirements over time. Consequently, the opportunity cost of excess debt financing can be measured as:

$$\text{Opportunity Costs} = \frac{S}{2}(r_L - r_S) \tag{2}$$

where r_L is the interest rate on long-term debt; and r_S is the yield on short-term securities.

Applying *EOQ* model to the cost tradeoff between Equations (1) and (2), *B&M* showed that the cost minimizing value of S is:

$$\text{Optimal Bond Issue Size} = S' = \sqrt{\frac{2FD}{r_L - r_S}} \tag{3}$$

Subsequent applications of this model in the context of bond issue size and timing have involved refinements to Equation (3) or relaxation of the assumptions that underlie it [2, 4, 6, 11, 12]. A constant element of all these refinements, however, is the measurement of the opportunity cost of excess debt as the differential between short-term and long-term interest rates.

While the intuition behind Equation (2) seems straightforward, inserting the relationship between short-term and long-term yields into Equation (3) places an important constraint on the model. It can yield meaningful results only if $r_L > r_S$. In other words, the model cannot accommodate yield curves with a zero or a negative slope. While the yield curve typically does have a positive slope, the presence of a flat or inverted yield curve in the financial markets cannot be considered a unique occurrence, particularly during the past 20 years. Because the applicability of the model is dependent upon the prevailing term structure of interest rates, a redefinition of the opportunity cost of excess debt financing is needed to restore the viability of the model as an analytical tool.

A RESPECIFICATION OF THE OPPORTUNITY COST OF EXCESS DEBT FINANCING

We view the decision to issue bonds in excess of current financing requirements as an investment decision. The impact of this investment decision on firm value should signal the true opportunity cost of excess debt. This value impact can be expressed in terms of the familiar *NPV* criterion:

$$NPV = \sum_{t=1}^{n} \frac{rM(1-T)}{(1+K_M)^t} + \frac{M}{(1+K_M)^n} - M \tag{4}$$

where r is the interest rate on the securities, T is the marginal corporate tax rate; M is the market value of the marketable securities acquired; K_M is the appropriate discount rate; and n is the maturity of the securities. If it is assumed that the securities are purchased and redeemed at par value, the *NPV* in Equation (4) depends on the value of K_M. Specifically, $NPV = 0$ when $K_M = r(1-T)$.

Modigliani and Miller [9] proved that the weighted average cost of capital for a levered firm is equal to:

$$K_L = K_u(1-\phi T) \tag{5}$$

where K_L is the appropriate discount rate for a levered firm; K_u is the discount rate for an unlevered firm; and ϕ is the percentage of a firm's capital structure accounted for by debt financing. In the context of the investment decision required by excess debt financing, r is the correct risk-adjusted value of K_u. Thus, Equation (5) can be expressed as:

$$K_M = r(1-\phi T) \quad (6)$$

where all the variables have been previously defined.

Equation (6) indicates that the value effect of the investment of excess debt financing in marketable securities depends on a firm's capital structure. This can be more easily demonstrated through a simple example. If Equation (4) is assumed to be a perpetuity and Equation (6) is substituted, Equation (4) reduces to:

$$NPV = \frac{r}{r}\frac{M(1-T)}{(1-\phi T)} - M = \frac{-r}{r}\frac{M(1-\phi)T}{(1-\phi T)} \quad (7)$$

The denominator in Equation (7) is the discount rate for the perpetuity, while the numerator is the net cash flow per period. This cash flow is the difference between the taxes paid on the interest revenues from corporate lending and the offsetting tax saving of any corporate borrowing used to finance that lending. At a 100% equity financing (ϕ=0), $NPV = -MT$. This negative value impact associated with lending is identical in size to the positive value impact associated with borrowing that was first pointed out by Modigliani and Miller [9]. As the proportion of debt financing increases, the negative value impact decreases, approaching zero only at the extreme case of 100% debt financing (ϕ=1).

Because the *EOQ* model is a single period model, the numerator of Equation (7) measures the negative cash flow per period associated with the decision to invest excess debt financing in marketable securities. This is the appropriate opportunity cost that should be of most concern. Since M in Equation (7) also represents the average level of excess debt financing [$S/2$ in Equation (2)], Equations (2) and (3) can be restated as follows:

$$\text{Opportunity Costs} = \frac{S}{2}\left[r\,(1-\phi)\ T\right] \quad (8)$$

$$\text{Optimal Bond Issues Size} = S^{*} = \sqrt{\frac{2FD}{r(1-\phi)T}} \quad (9)$$

These two equations reveal that both the proportion of debt financing utilized by a firm (ϕ) and the firm's marginal tax rate (T) play an important role in determining the optimal size and timing of a firm's bond issues.

IMPLICATIONS OF THE RESPECIFIED MODEL

To illustrate the effect of this respecification of the opportunity cost consider a firm that will need $50 million in financing over the next three years. Assume that the fixed component of flotation costs is $100,000 and that the yield-to-maturity on short-term and long-term securities is currently 12% and 14%, respectively. Since the planning horizon is three years, the denominator in Equation (3) takes on the following value:

$$(r_L - r_S)\ (3) = (.14 - .12)\ (3) = .06$$

Substitution of these values into Equation (3) yields:

$$S' = \sqrt{\frac{2(\$100{,}000)(\$50{,}000{,}000)}{.06}} = \$12{,}909{,}270$$

This bond issue size implies that the firm should issue debt every 9.3 months (i.e., 36 months/($50,000,000/$12,909,270 = 9.3).

The firm's financing requirements can yield either identical or very dissimilar results in the respecified model depending on the firm's capital structure and its marginal tax rate. For example, the optimal bond issue size indicated by Equation (9) will approximate that of Equation (3) if the firm's capital structure consists of 58% debt financing and its marginal tax rate is 34%:

$$S^* = \sqrt{\frac{2(\$100{,}000)(\$50{,}000{,}000)}{.14(1-.58)\ (.34)\ (3)}} = \$12{,}912{,}527$$

Use of an r of 14% is consistent with the expectations hypothesis of the yield curve which requires that the average yield on the series of short-term securities to be acquired by the firm over time be identical to the initial yield on the long-term debt issued by the firm. The effect of capital structure and marginal tax rate in the respecified model is demonstrated in Exhibits 1 and 2. The first exhibit shows, for a marginal tax rate of 34%, the effect of different capital structures on optimal bond size, frequency of bond issues, and total number of debt issues over the firm's three-year planning horizon. It also shows the effect of capital structure on the optimal size bond issue indicated by Equation (9) relative to the optimum indicated by (3). The second exhibit indicates, for a capital structure of 50% debt financing, the effect of different marginal tax rates on the results produced by Equation (9).

EXHIBIT 1

The Effect of Capital Structure in the Respecified Model

T = .34

Debt Financing (ϕ)	S^*	S^*/S'	Frequency of Debt Issues (months)	Number of Debt Issues
0	$8,368,274	.65	6.0	6.0
.25	9,662,851	.75	7.0	5.1
.50	11,834,526	.92	8.5	4.2
.58	12,921,527	1.00	9.3	3.9
.75	16,736,546	1.30	12.1	3.0
.90	26,462,806	2.05	19.1	1.9

S*: Optimal Bond Issue Size, Respecified Model
S': Optimal Bond Issue Size, Prior Model

EXHIBIT 2

The Effect of Tax Rate in the Respecified Model

ϕ = .50

Marginal Tax Rate (T)	S^*	S^*/S'	Frequency of Debt Issues (months)	Number of Debt Issues
.10	$21,821,788	1.69	15.7	2.3
.15	17,817,415	1.38	12.8	2.8
.20	15,430,334	1.20	11.1	3.2
.25	13,801,310	1.07	9.9	3.6
.30	12,598,815	.98	9.1	4.0
.35	11,664,237	.90	8.4	4.3
.40	10,910,894	.85	7.9	4.6

S*: Optimal Bond Issue Size, Respecified Model
S': Optimal Bond Issue Size, Prior Model

The data in Exhibits 1 and 2 indicate how sensitive bond issue size is in the respecified model to both a firm's capital structure and its marginal tax rate. At a marginal tax rate of 34%, the optimal bond issue size indicated by Equation (9) varies from a low of 65% to a high of 205% of the optimum indicated by Equation (3). This sensitivity of optimal debt issue size to capital structure is a direct result of the opportunity cost of excess financing indicated by Equation (8). The net disadvantage to a firm of being a lender can be offset by funding those loans with debt financing. Consequently, the lower opportunity cost of excess debt for highly levered firms should encourage them to issue larger amounts of debt less frequently than less levered firms. A firm's tax status also plays an important role because it affects the size of the disadvantage to a firm of being a lender. As marginal tax rates increase, the opportunity cost of excess debt financing given by Equations (8) also increases.

CONCLUSIONS

The redefinition of the opportunity cost of excess debt financing in this paper corrects the flaw in prior models that made their usefulness dependent on the slope of the yield curve at a given moment in time. The model developed in this paper can accommodate a yield curve of any shape because its definition of the opportunity cost of excess debt financing does not incorporate the term structure of interest rates. Instead, this opportunity cost is measured in a manner that is consistent with the Modigliani-Miller hypothesis concerning the effect of borrowing on firm value. By extending this hypothesis to the case of firm lending, we have shown that the opportunity cost of excess financing depends on a firm's capital structure and marginal tax rate. Thus, firms facing identical financing requirements may have very differential optimal strategies regarding the size and timing of their bond issues.

REFERENCES

1. Bierman, Jr., H. and A. K. McAdams, "Financial Decisions and New Decision Tools," *Financial Executive*, May 1964, pp. 23-30.

2. Bierman, Jr., H., "The Bond Issue Size Decision," *Journal of Financial and Quantitative Analysis*, December 1966, pp. 1-14.

3. Brealey, R. and S. Myers, *Principles of Corporate Finance*, Second Edition, New York, McGraw-Hill, 1984.

4. Dyl, E. and W. Sawaya, "The Bond Issue Size Decision Revisited," *Financial Management*, Winter 1979, pp. 60-67.

5. Hicks, J. R., *Value and Capital: An Inquiry Into Some Fundamental Principles of Economic Theory*, Second Edition, Oxford, Oxford University Press, 1946.

6. Litzenberger, R. H. and D. R. Rutenberg, "Size and Timing of Corporate Bond Flotation," *Journal of Financial and Quantitative Analysis*, January 1972, pp. 1343-1359.

7. Lutz, F. A. and V. C. Lutz, *The Theory of Investment in the Firm*, Princeton, NJ, Princeton University Press, 1951.

8. Miller, M., "Debt and Taxes," *Journal of Finance*, May 1977, pp. 261-275.

9. Modigliani, F. and M. Miller, "Corporate Income Taxes and the Cost of Capital: A Correction," *American Economic Review*, June 1963, pp. 433-443.

10. Myers, S. C., "Interactions of Corporate Financing and Investment Decisions-Implications for Capital Budgeting," *Journal of Finance*, March 1974, pp. 1-25.

11. Van Horne, J. C., "Implied Fixed Costs in Long-Term Debt Issues," *Journal of Financial and Quantitative Analysis*, December 1973, pp. 821-834.

12. Van Horne, J. C., *Financial Management and Policy*, Sixth Edition, Englewood Cliffs, NJ, Prentice-Hall, Inc., 1983, pp. 570-573.

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